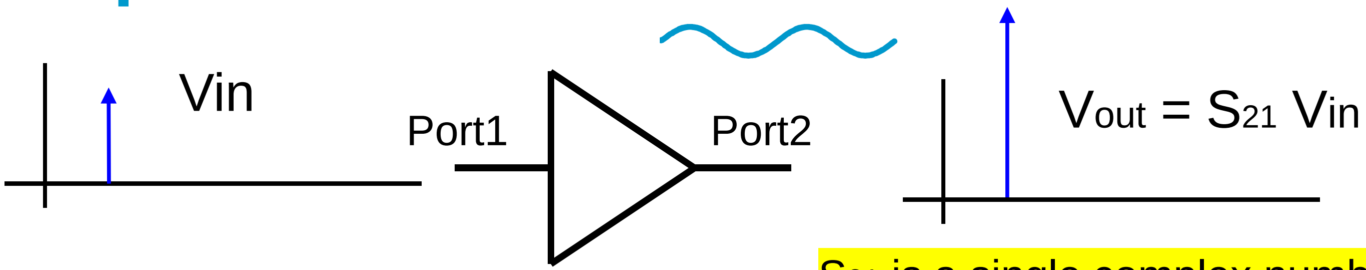
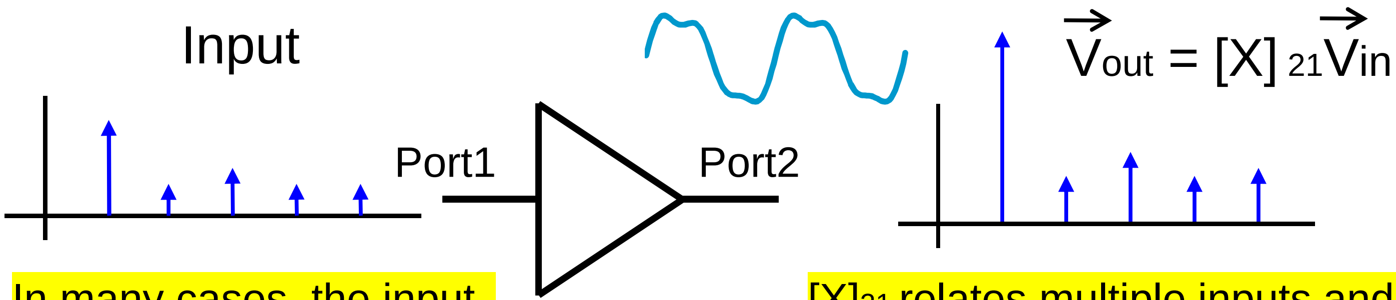
S-parameters can handle one input frequency and the same output frequency

X-parameters can handle multiple input and output frequencies.



$$S = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix}$$

S_{21} is a single complex number.



$$X = \begin{bmatrix} [X]_{11} & [X]_{12} \\ [X]_{21} & [X]_{22} \end{bmatrix}$$

In many cases, the input signal also has distortion.

$[X]_{21}$ relates multiple inputs and multiple outputs, thus, it's a matrix

What Exactly Are X-Parameters???

- Two words: **behavioral models!**
- Completely describe a device's nonlinear performance
- Include the magnitude and phase of the fundamental signal, **all of its harmonics and intermodulation products**, and all of their dependence on source and load impedance, bias, etc.
- Are ***cascadable*** like S-parameters

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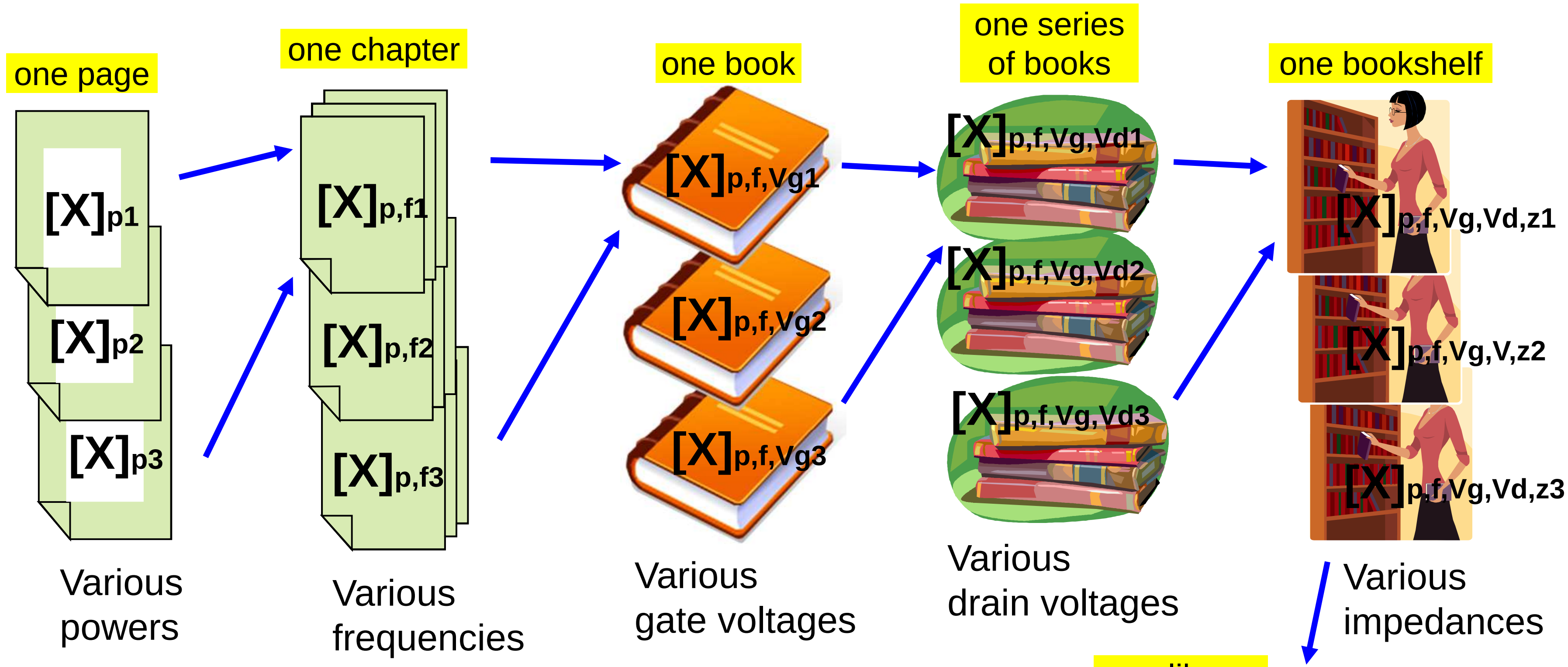
Why Are X-Parameters Revolutionary?

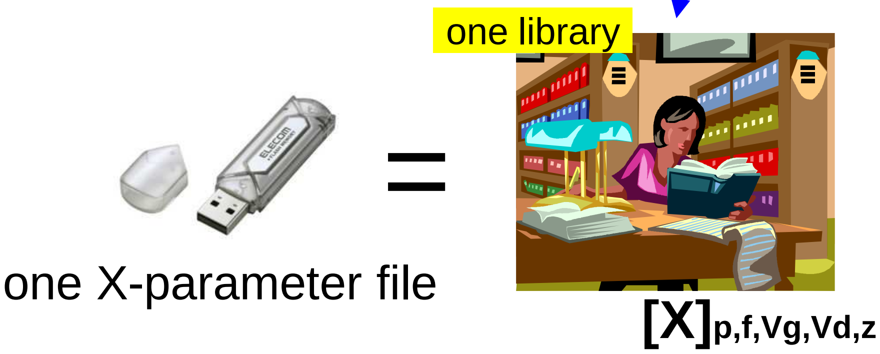
- Provide predictable measurement-based nonlinear design
- Generate nonlinear models much faster than traditional methods
- X-parameters, ADS, and NVNA are used to:
  - Reconstruct time-domain waveforms
  - Estimate performance parameters such as ACPR, EVM, and PAE
  - Design multi-stage amplifiers and subsystems
  - Optimize nonlinear system performance
- Less design iterations required, resulting in shorter design cycle

Business value: faster time-to-market!

X-parameters: Large Data Library with Many Variables



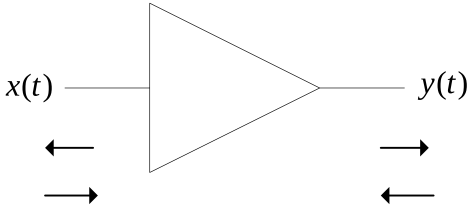
Measurement takes from tens of minutes to a several hours depending on the size of the “library”

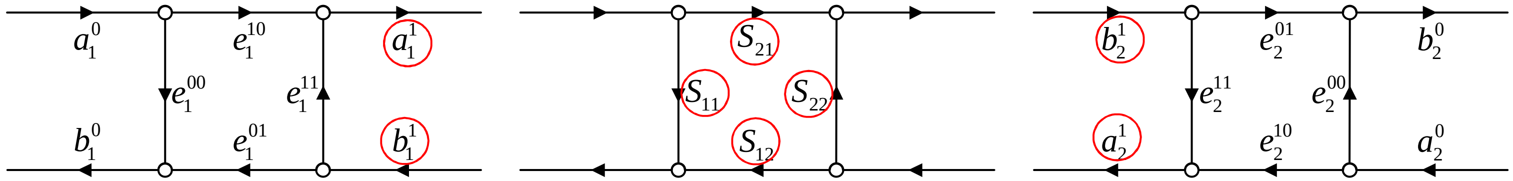


Scattering Parameters – Linear Systems

- **Linear Describing Parameters**

Linear S-parameters by definition require that the S-parameters of the device do not change during measurement.





$$b_1 = S_{11}a_1 + S_{12}a_2$$

$$b_2 = S_{21}a_1 + S_{22}a_2$$

$$\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$$

S-Parameter Definition

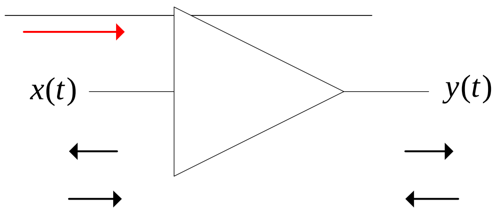
To solve VNA's traditionally use a forward and reverse sweep (2 port error correction).

Scattering Parameters – Linear Systems

- **Linear Describing Parameters**

If the S-parameters change when sweeping in the forward and reverse directions when performing 2 port error correction then by definition the resulting computation of the S-parameters becomes invalid.

$$b_1 = S_{11}a_1 + S_{12}a_2$$
$$b_2 = S_{21}a_1 + S_{22}a_2$$



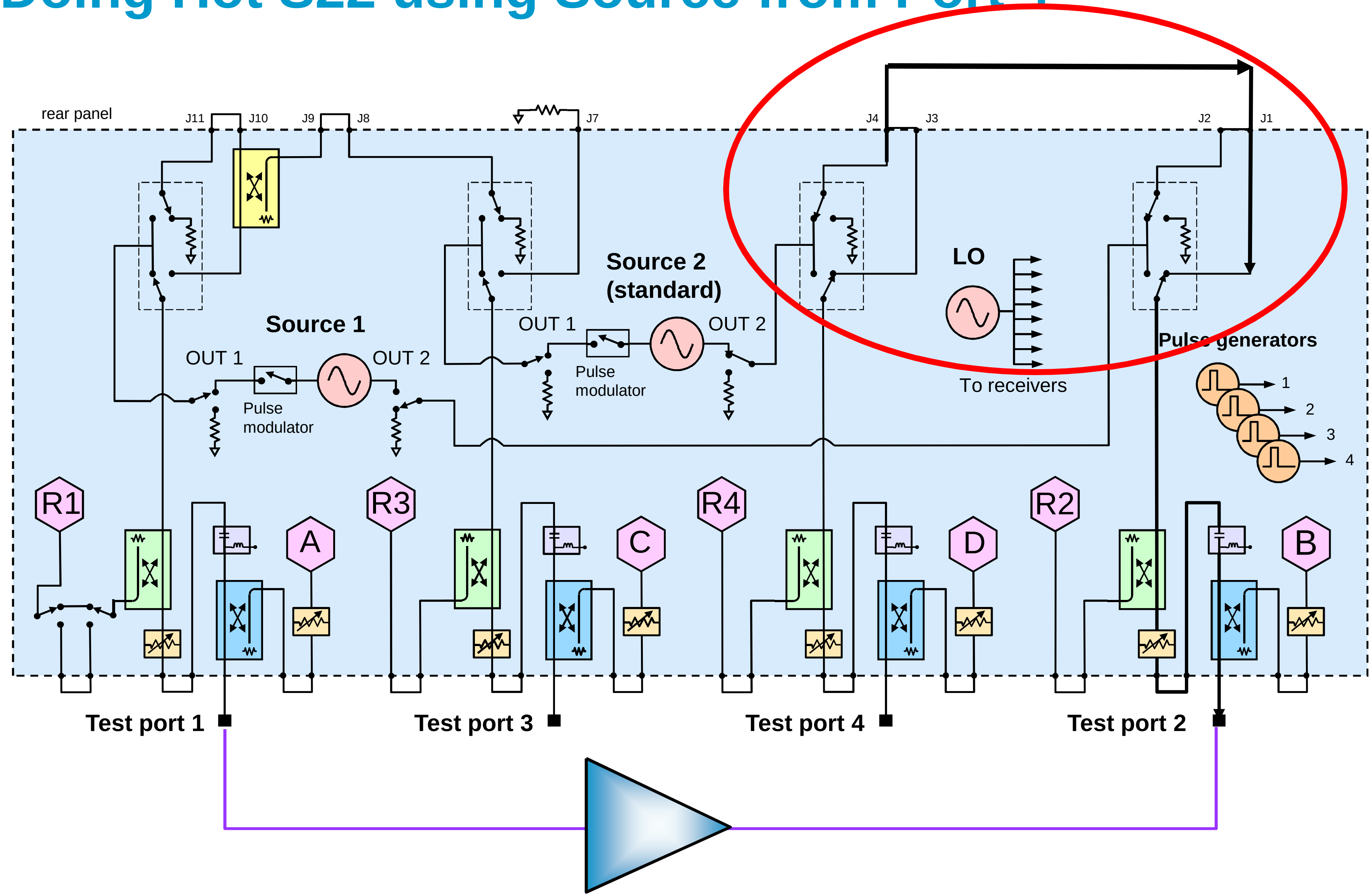
$$\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$$

Hot S22 (|a1|)

This is often why people are asking for Hot S22 because the match is changing versus <u>input drive power</u> and frequency (Nonlinear phenomena). Hot S22 traditionally measured at a frequency slightly offset from the large input drive signal.

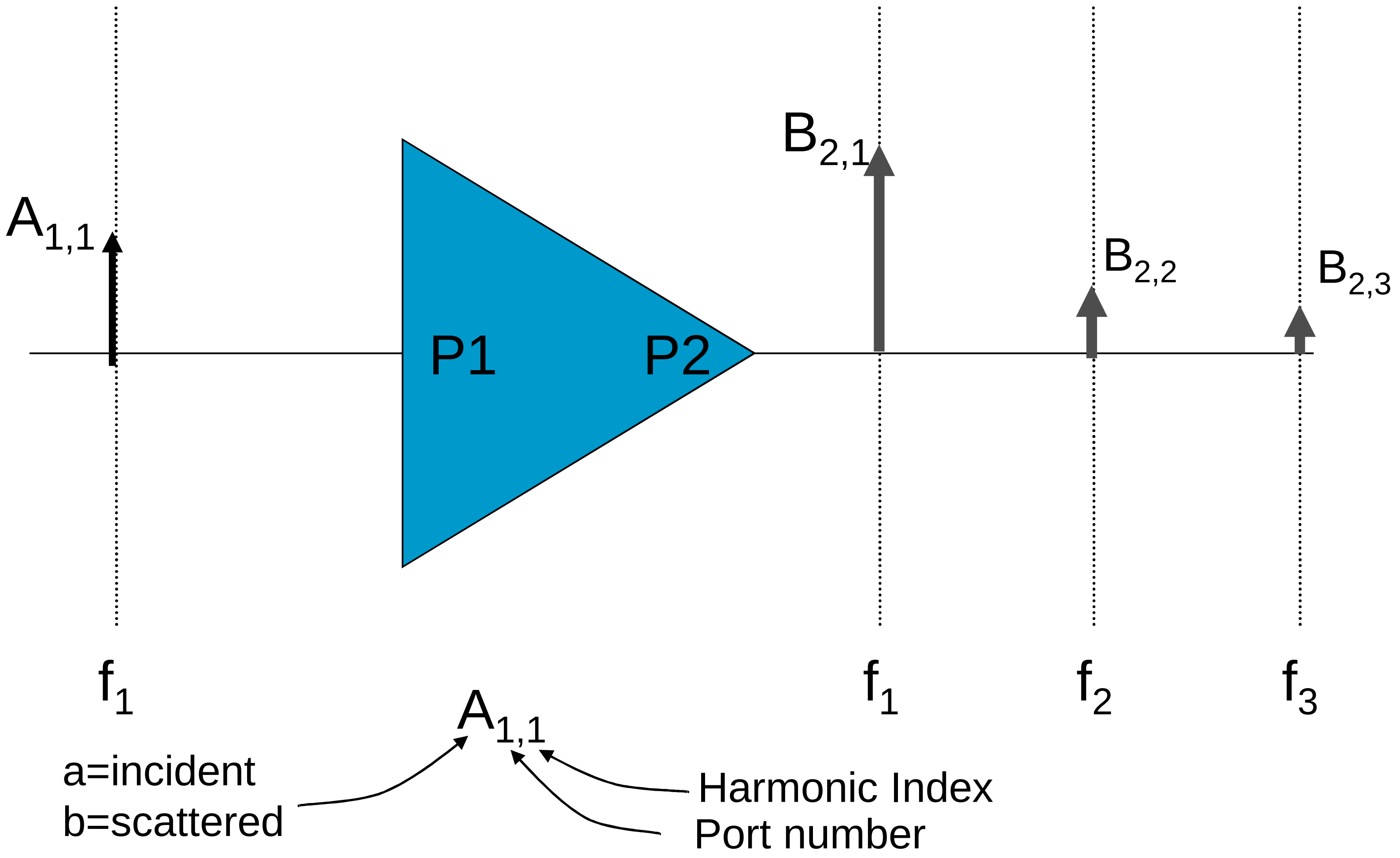
$$\begin{bmatrix} b_1^f & b_1^r \\ b_2^f & b_2^r \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \begin{bmatrix} a_1^f & a_1^r \\ a_2^f & a_2^r \end{bmatrix}$$

Doing Hot S22 using Source from Port 4

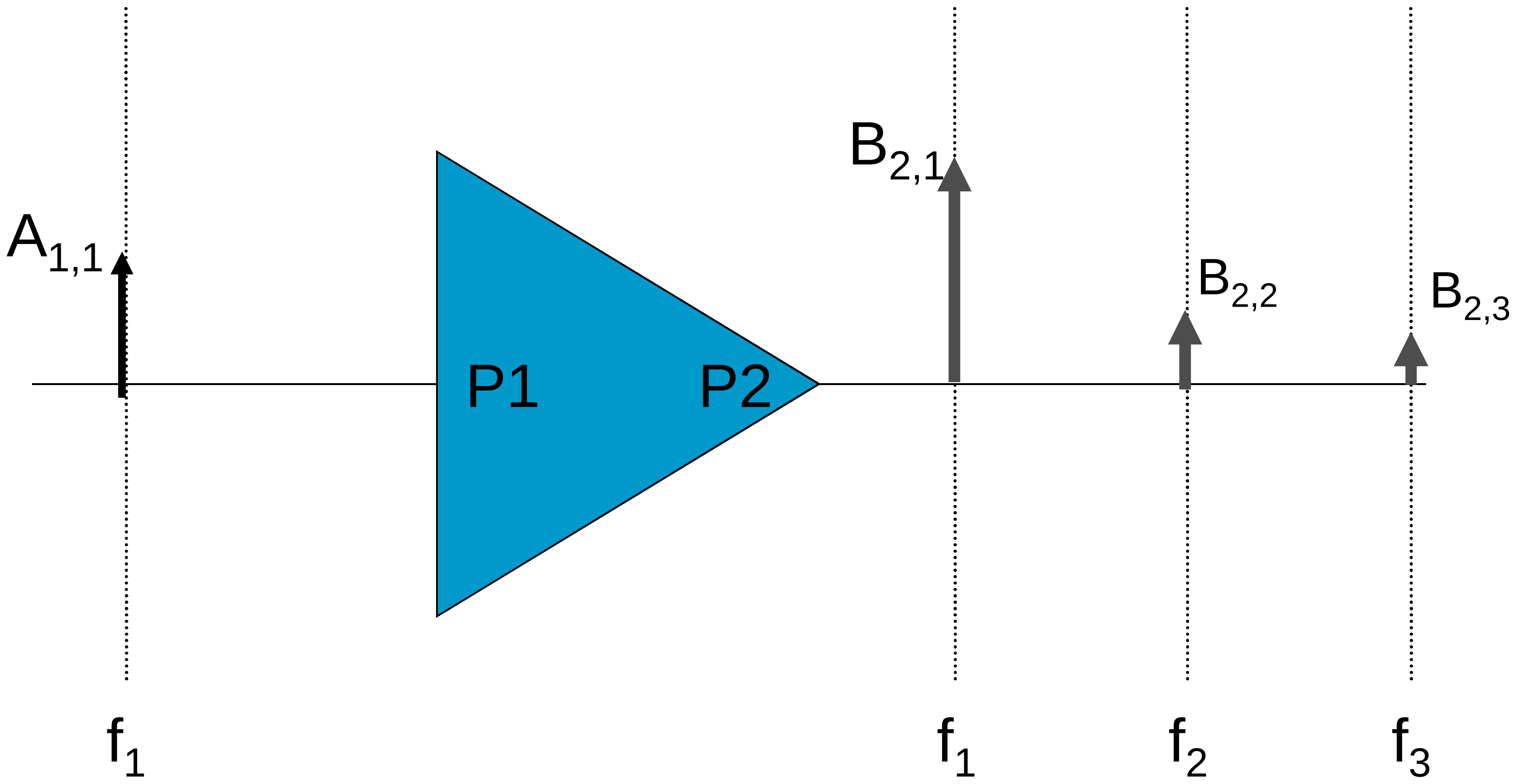


Lets see what really happens with hot S22:

Start by driving a signal into the amplifier



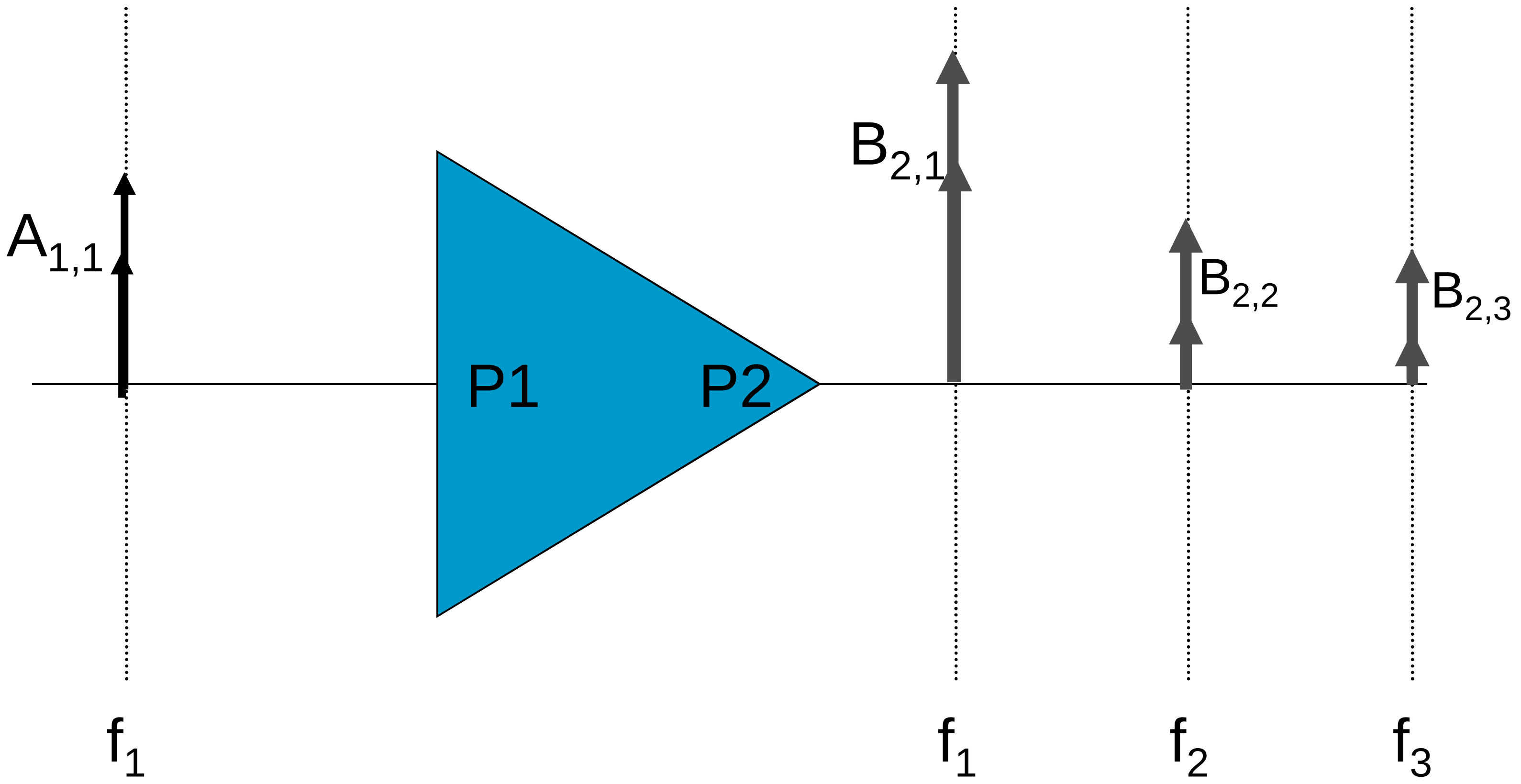
Lets see what really happens with hot S22



We call the "B" response $\mathbf{X^F(|A_{11}|)}$
It is the output response to an input,
as a function of the input amplitude.

Lets see what really happens with hot S22

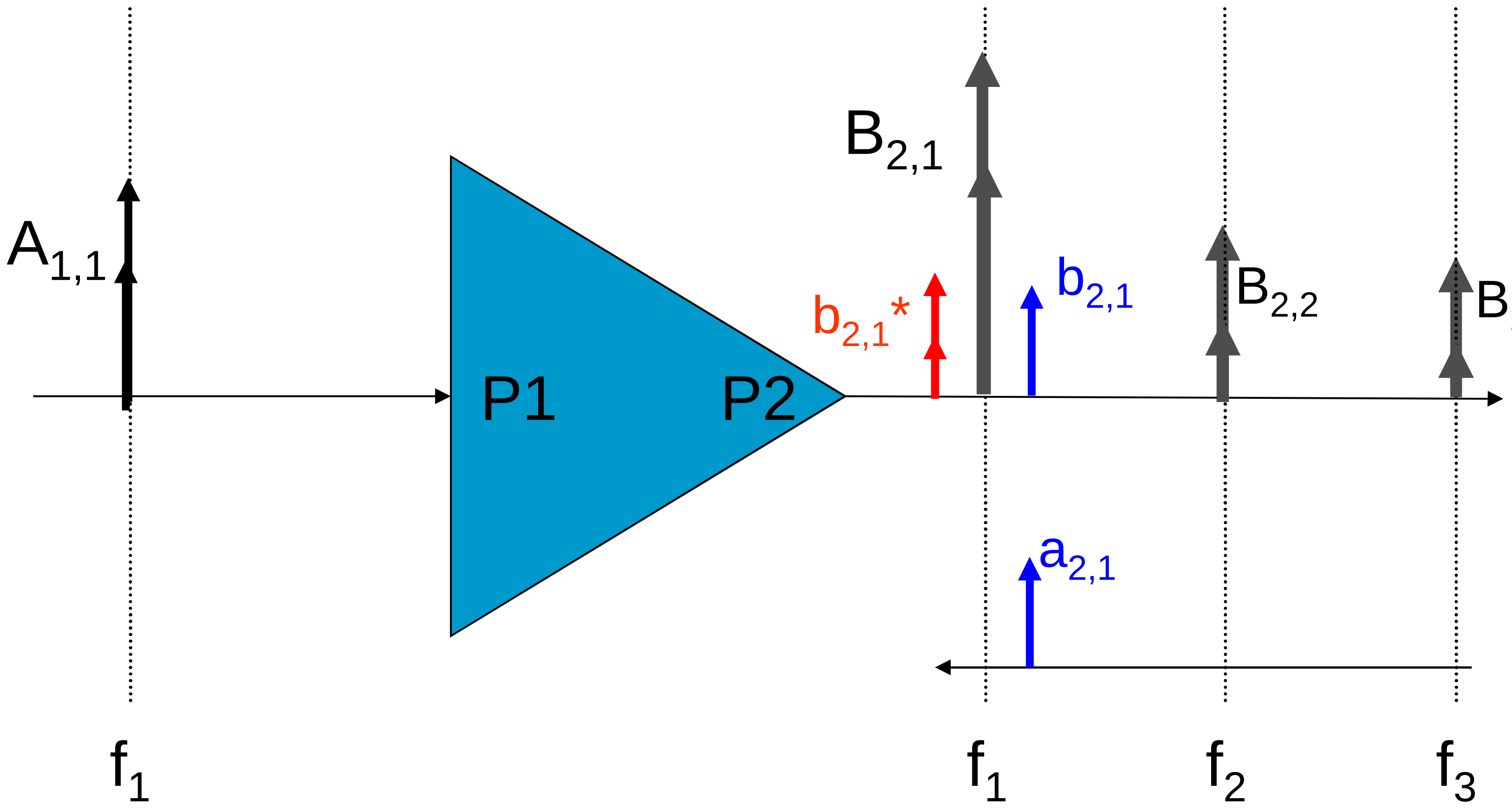
Now, lets make the input signal bigger



The harmonics increase faster than the fundamental, generally, by their order number (2nd = twice as fast)

Lets see what really happens with hot S22

Let's add a small signal incident on port 2, offset in freq



Normally, we would say S22=b2/a2

What term describes b2*? b2* is ***T***ransposed on the other side of B2

Hint

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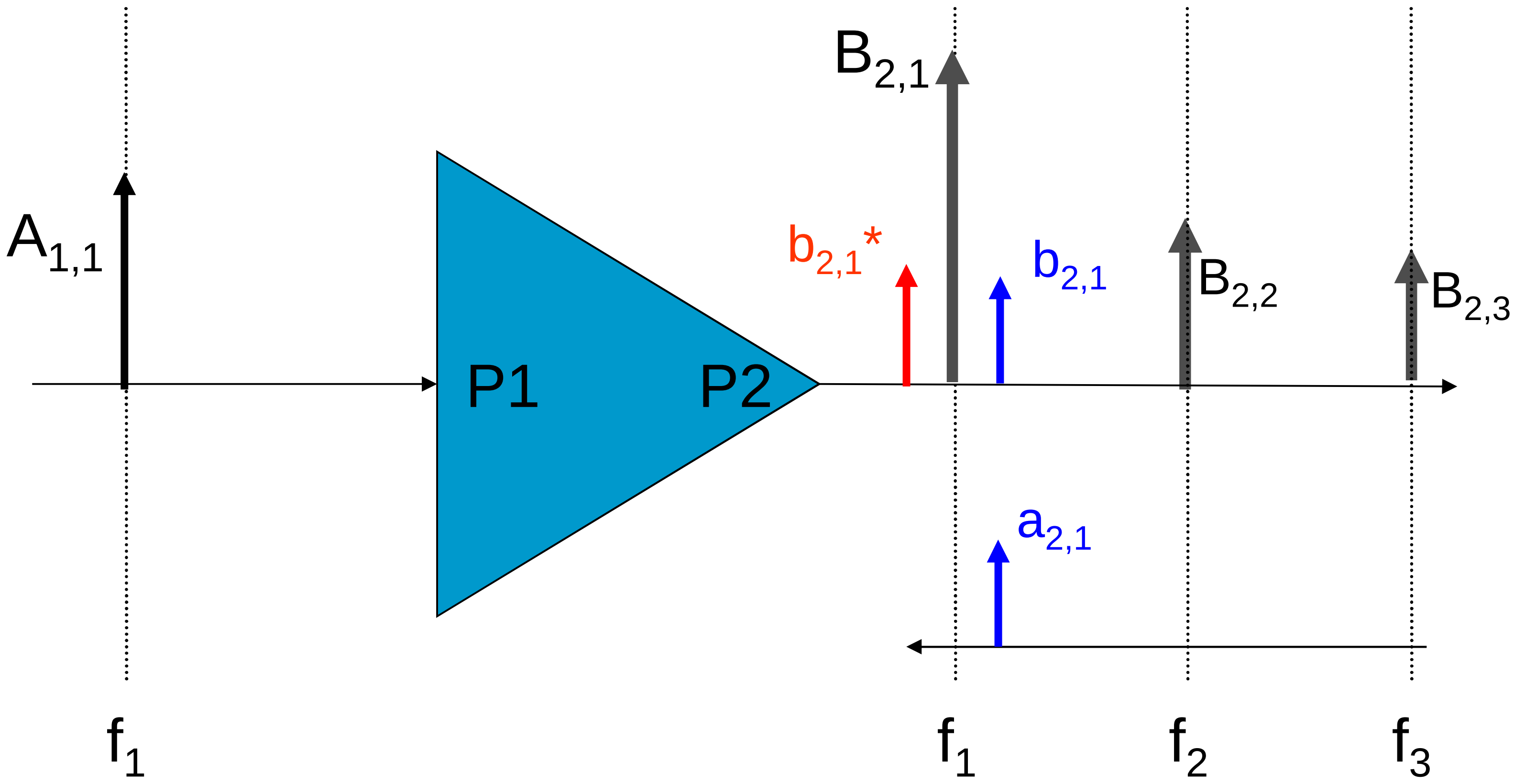
Lets see what really happens with hot S22

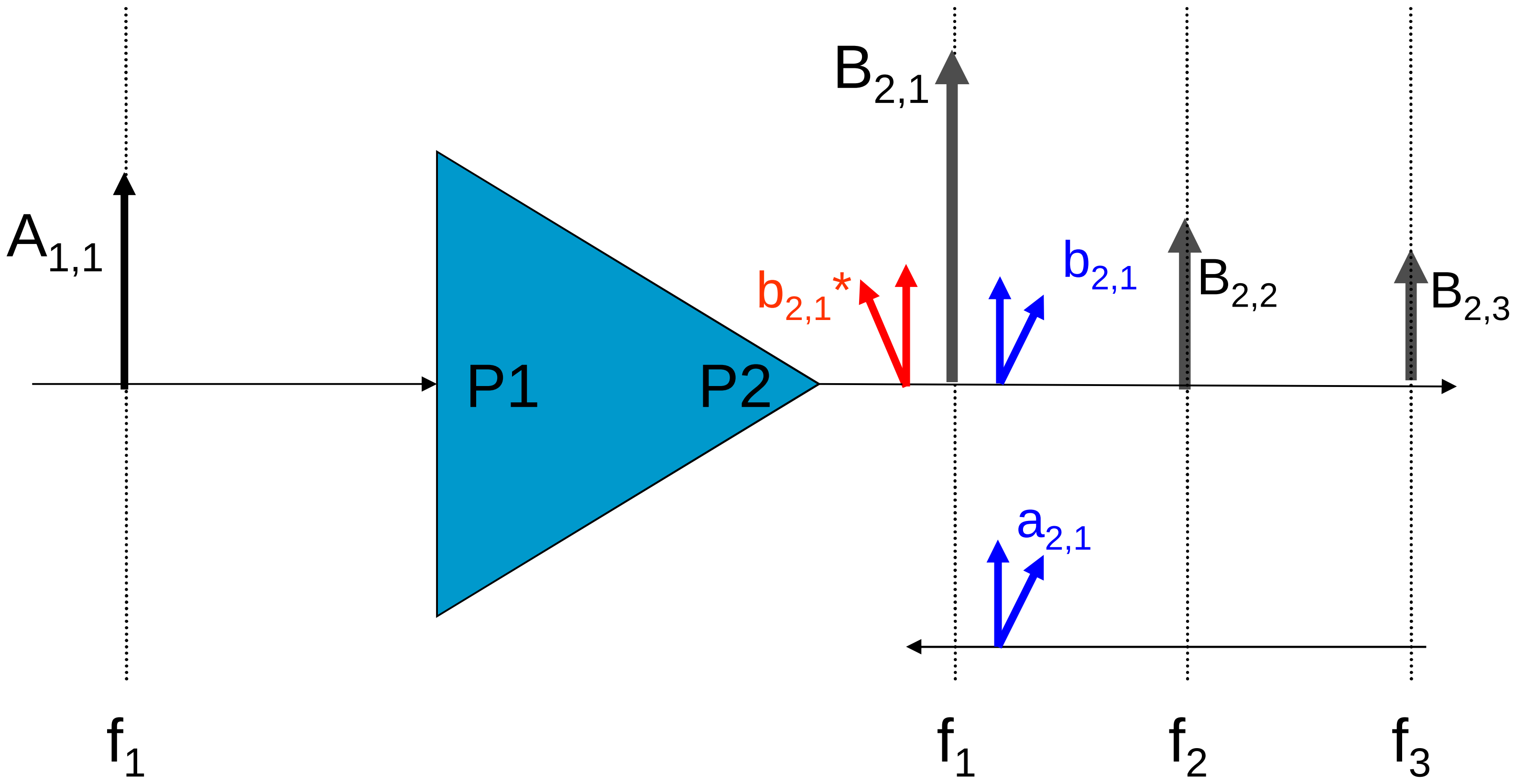
Let's change the frequency of a21



When a2 goes up in frequency, b2* goes ***down*** in frequency

Lets see what really happens with hot S22

Let's change the phase of a21



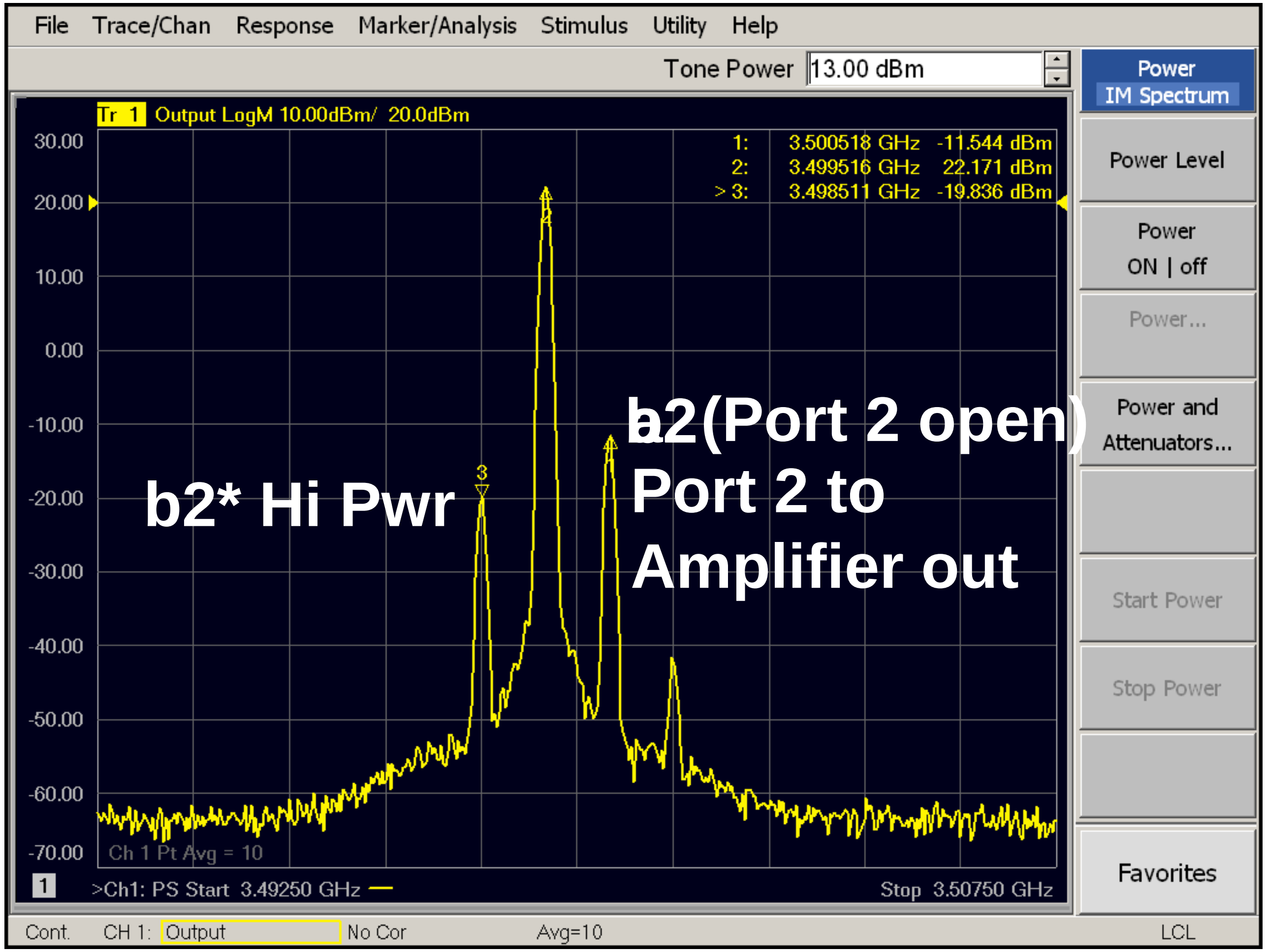
b2* moves as the conjugate of a2 phase

What do we call a term that changes with opposite phase? a **CONJUGATE** term!

b2*=T22·a2* Now we can see that T22 = b2*/a2*

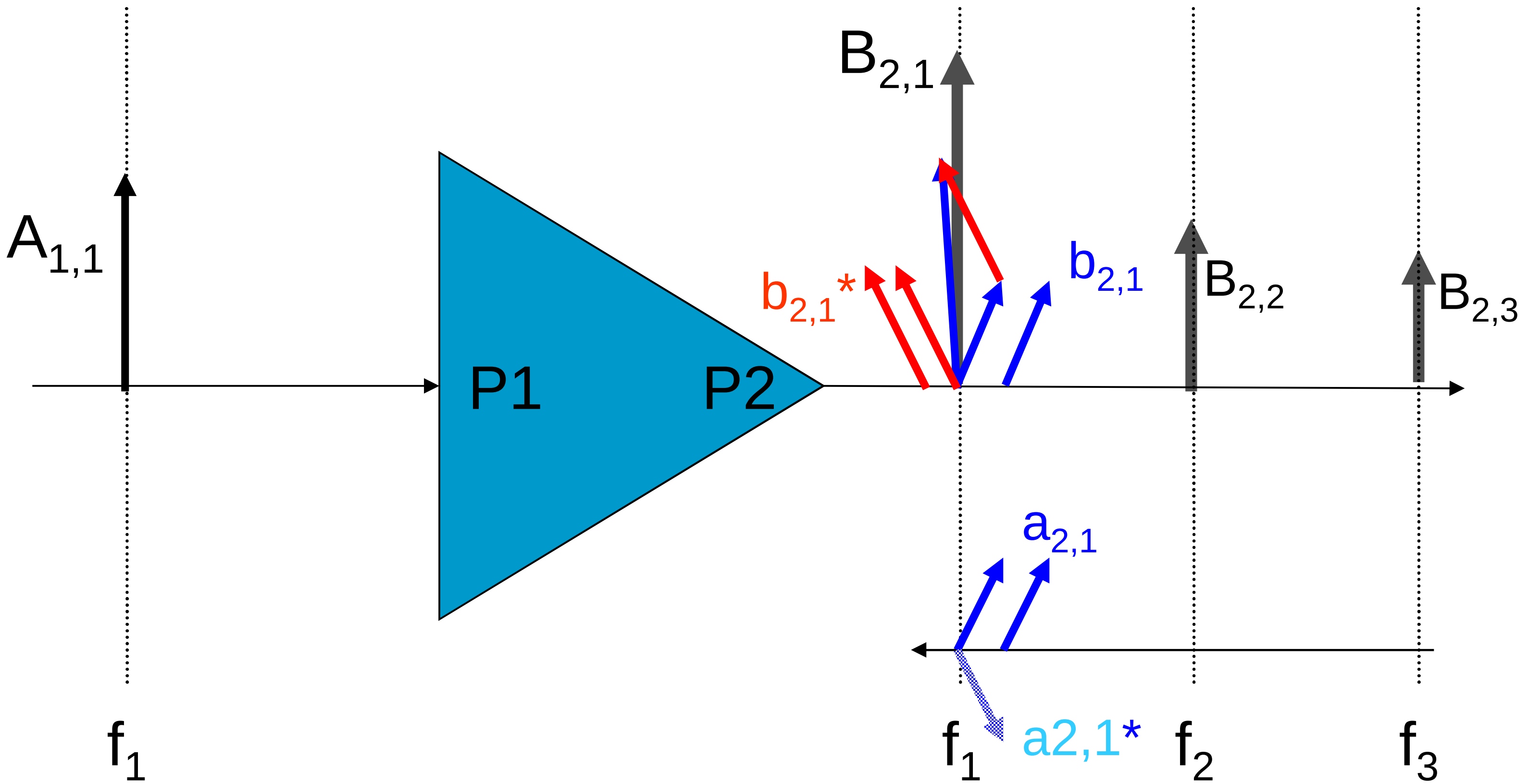
And it is CLEAR that HOT S22 is insufficient to describe the reflection behavior

We can see this real time on the PNA-X



Lets see what really happens with hot S22

Now what happens if a2 is not offset in freq?

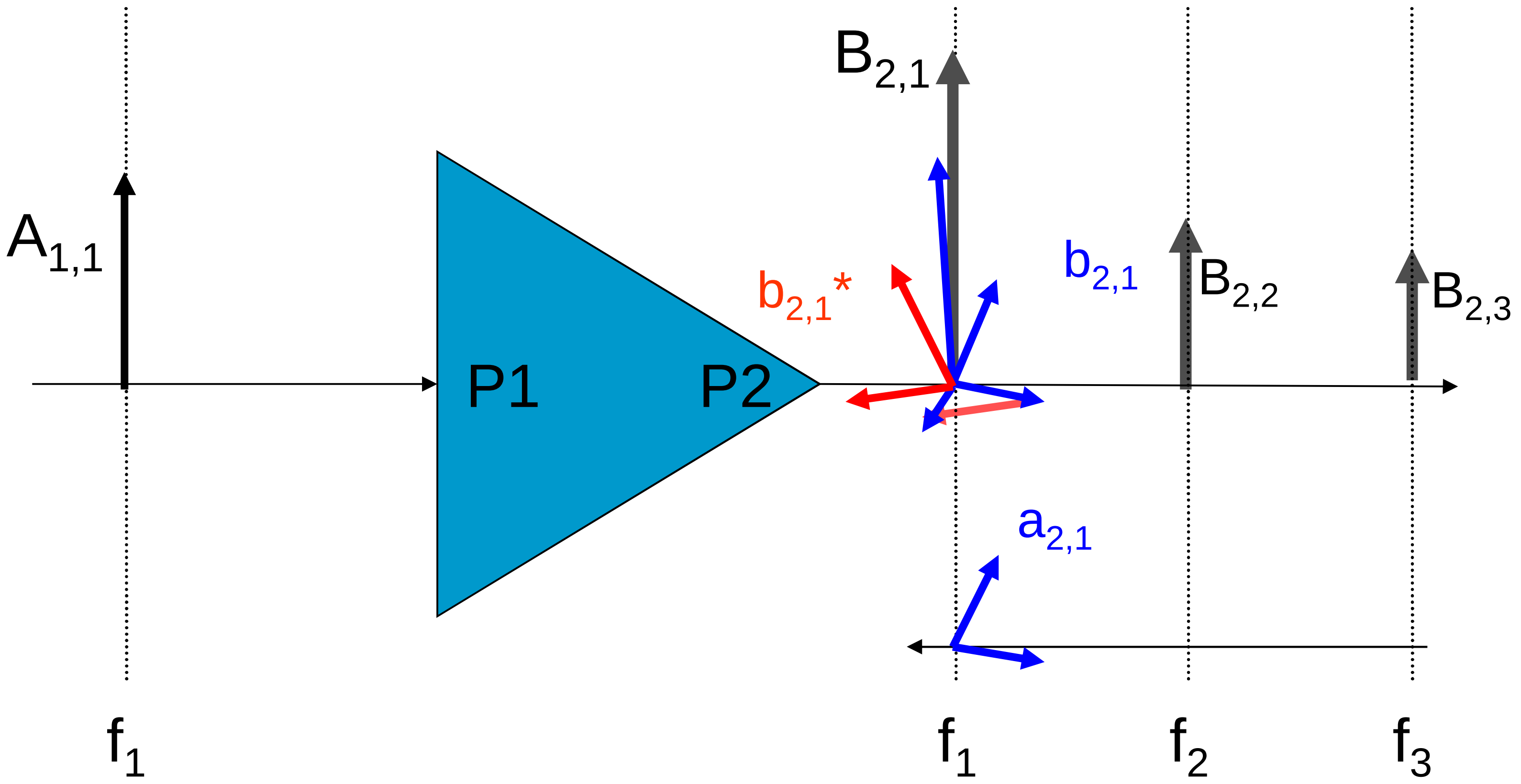


The total response of the b2 scattered signal, due to a2 is a combination of b2 and b2*: b2=S22 x a2 + T22 x a2*

$b2 = X^{S}_{22}(|A_{11}|)a2 + X^{T}_{22}(|A_{11}|)a2^*$; $\mathbf{b2=X_{22}(|A_{11}|)a2}$

Lets see what really happens with hot S22

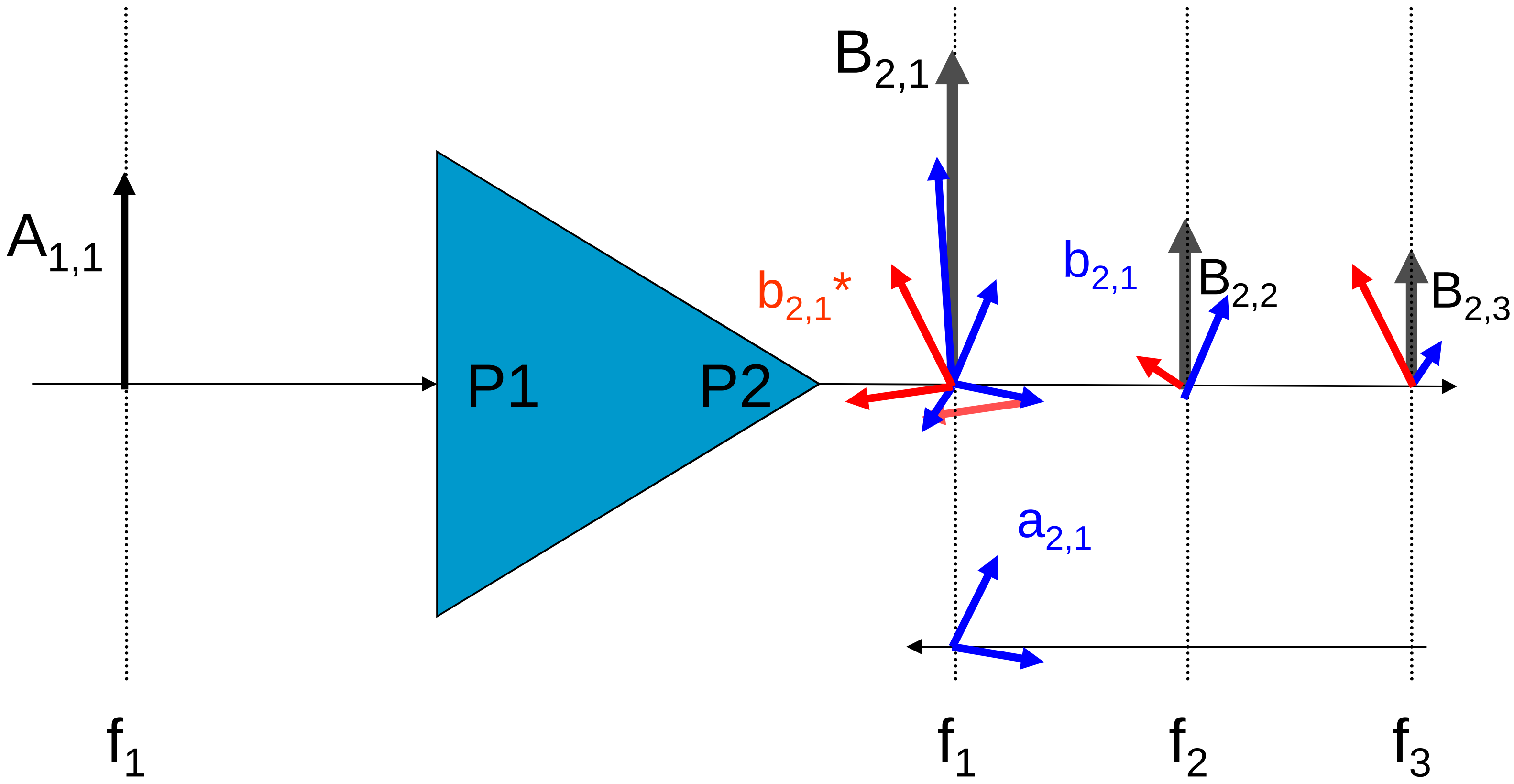
Now what happens if a2 is not offset in freq?



Changing the phase of a2 changes the total magnitude of b2 wave, but not the magnitude of individual parts, b2 and b2*

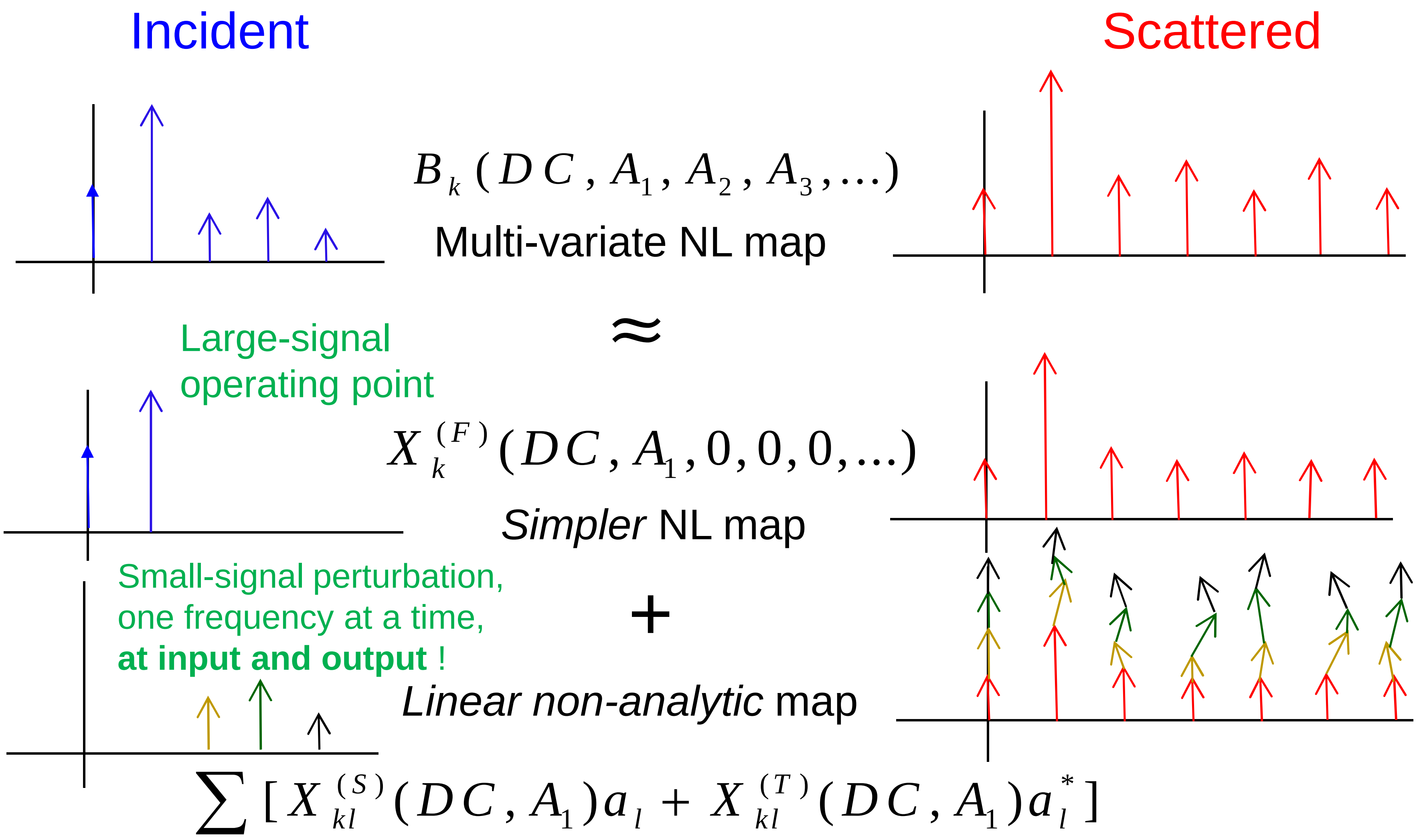
Lets see what really happens with hot S22

Now what happens if a2 is not offset in freq?



Changing the phase of a2 changes the total magnitude of b2 wave, but not the magnitude of individual parts, b2 and b2*

X-parameter Concept:



X-Parameter Extraction

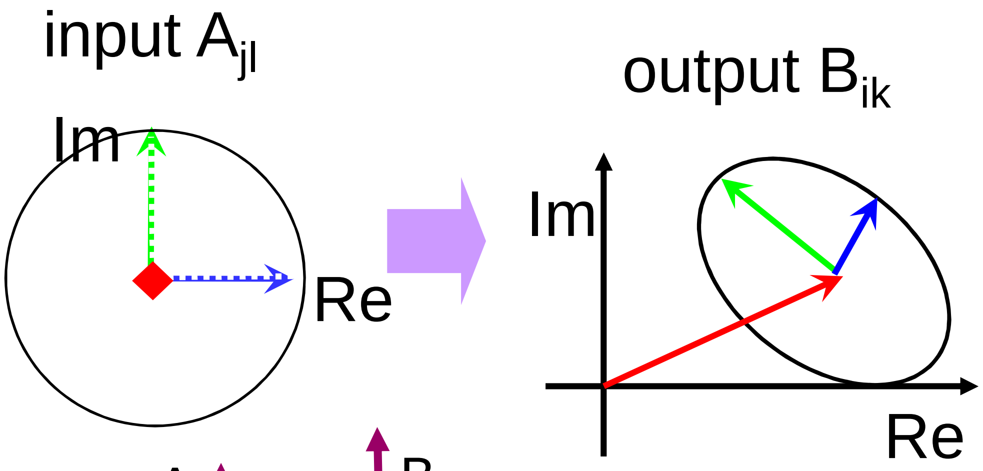
Kind of Active Load/Source Pull

Large-signal 50 Ω operating point

Extraction tone measures match dependency using two phase conditions

$$B_{ik} = X_{ik}^{(F)}(|A_{11}|)P^k + X_{ik,jl}^{(S)}(|A_{11}|)P^{k-l}a_{jl} + X_{ik,jl}^{(T)}(|A_{11}|)P^{k+l}a_{jl}^*$$

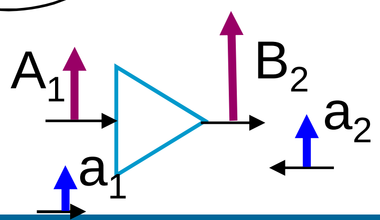
Perform 3 independent experiments with fixed A_1 using orthogonal phases of a_2



$$B_{ik}^{(0)} = X_{ik}^{(F)}(|A_{11}|)P^k$$

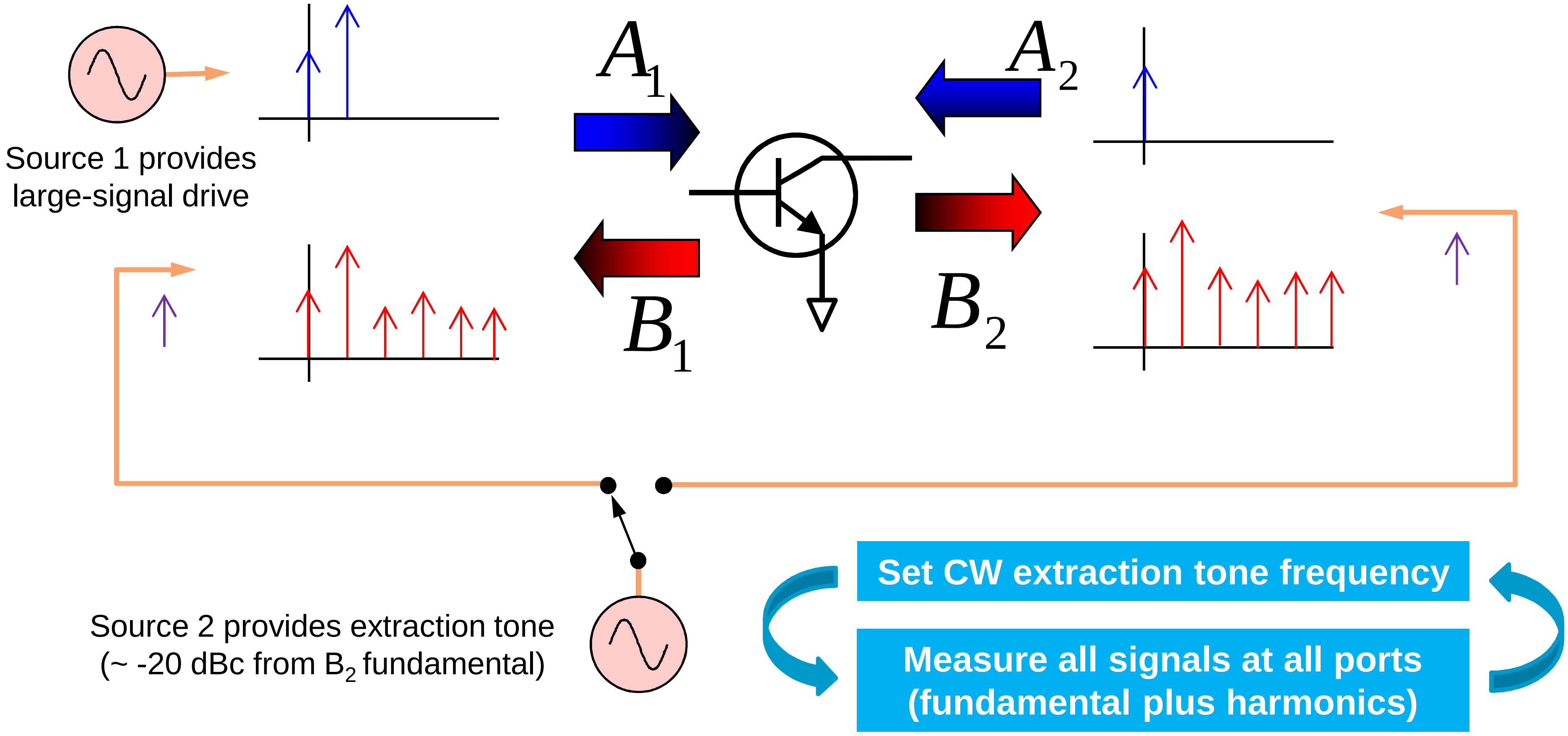
$$B_{ik}^{(1)} = X_{ik}^{(F)}(|A_{11}|)P^k + X_{ik,jl}^{(S)}(|A_{11}|)P^{k-l}A_{jl}^{(1)} + X_{ik,jl}^{(T)}(|A_{11}|)P^{k+l}A_{jl}^{(1)*}$$

$$B_{ik}^{(2)} = X_{ik}^{(F)}(|A_{11}|)P^k + X_{ik,jl}^{(S)}(|A_{11}|)P^{k-l}A_{jl}^{(2)} + X_{ik,jl}^{(T)}(|A_{11}|)P^{k+l}A_{jl}^{(2)*}$$



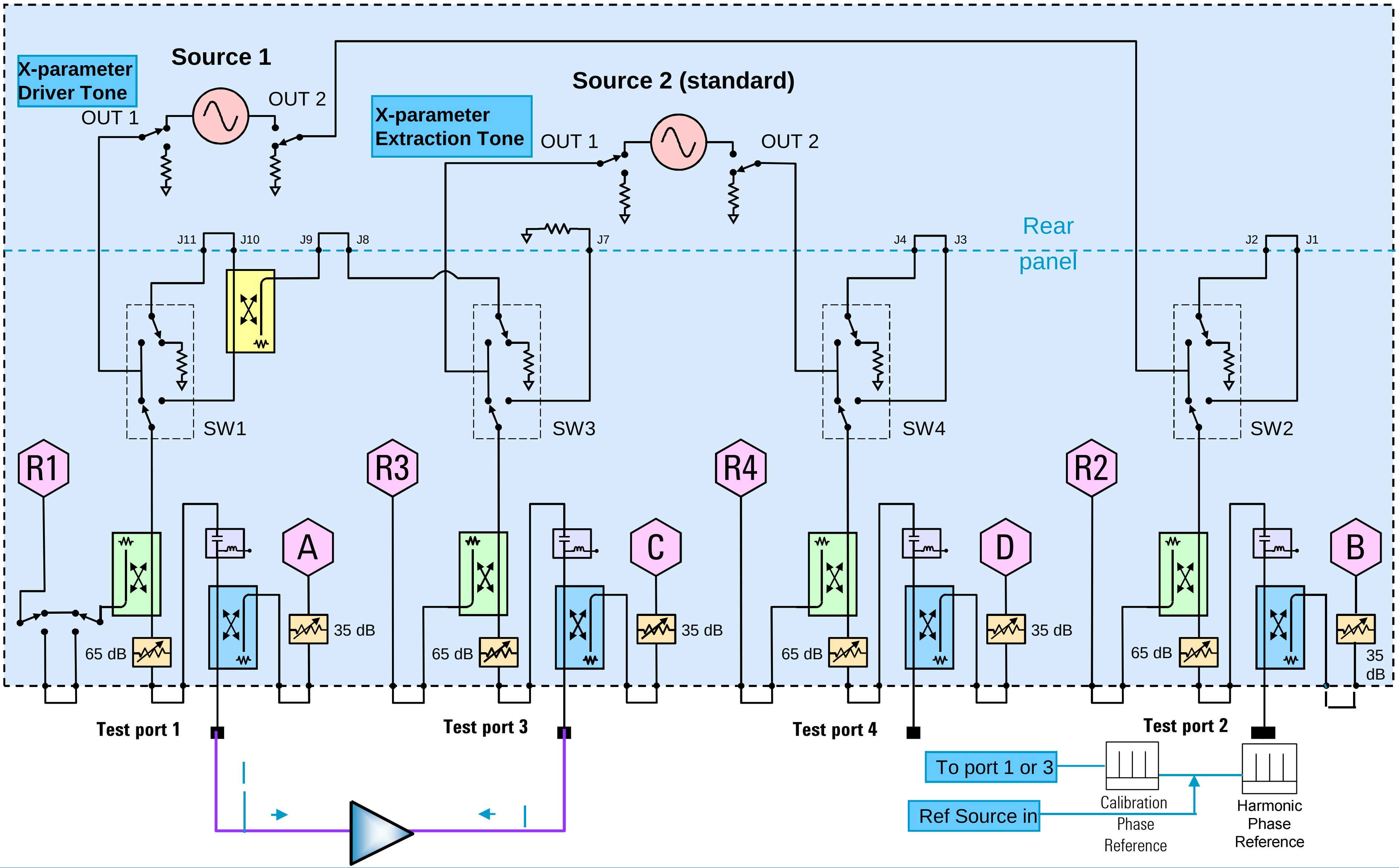
For output port i, output harmonic k; input port j, input harmonic l

Extraction Tone Provides Small-Signal Perturbation For Each Harmonic



Repeat extraction-tone loop for each large-signal drive level, frequency, bias, etc.

X-parameter Physical Measurements



Scattering Parameters

S-Parameters – Linear System Description

$$b_i = \sum_k S_{ik} \cdot a_k$$

$$b_1 = S_{11} a_1 + S_{12} a_2$$
$$b_2 = S_{21} a_1 + S_{22} a_2$$

X-Parameters – Linear and Nonlinear System Description

$$b_{ij} = X_{ij}^{(F)}(|A_{11}|) P^{j} + \sum_{k,l \neq (1,1)} \left(X_{ij,kl}^{(S)}(|A_{11}|) P^{j-l} \cdot a_{kl} + X_{ij,kl}^{(T)}(|A_{11}|) \; P^{j+l} \cdot a_{kl}^{*} \right)$$

$|A_{11}|$ = Large signal drive to the amplifier input port (port #1) at the fundamental frequency (#1)

Definitions

- **i = output port index**
- **j = output frequency index**
- **k = input port index**
- **l = input frequency index**

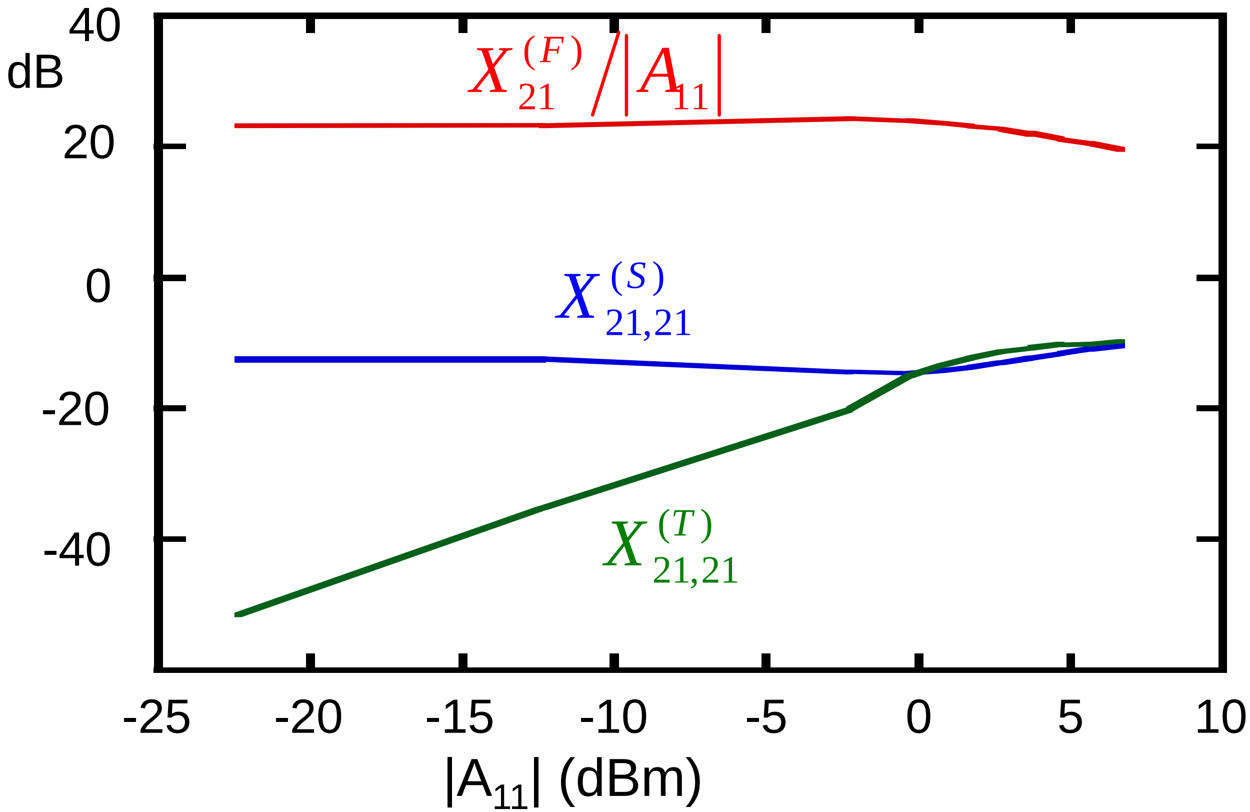
For example: $X_{21,21}^{T}$

Means: output port = 2
output frequency = 1 (fundamental)
input port = 2
input frequency = 1 (fundamental)

X-parameters Reduce to S-parameters

$$B_{11}(|A_{11}|) = X_{11}^{(F)}(|A_{11}|)P + X_{11,21}^{(S)}(|A_{11}|)A_{21} + X_{11,21}^{(T)}(|A_{11}|)P^2 A_{21}^*$$

$$B_{21}(|A_{11}|) = X_{21}^{(F)}(|A_{11}|)P + X_{21,21}^{(S)}(|A_{11}|)A_{21} + X_{21,21}^{(T)}(|A_{11}|)P^2 A_{21}^*$$



$$X_{11}^{(F)}/|A_{11}| \underset{|A_{11}|\to 0}{\to} s_{11}$$

$$X_{21}^{(F)}/|A_{11}| \underset{|A_{11}|\to 0}{\to} s_{21}$$

$$X_{11,21}^{(S)}(|A_{11}|) \underset{|A_{11}|\to 0}{\to} s_{12}$$

$$X_{21,21}^{(S)}(|A_{11}|) \underset{|A_{11}|\to 0}{\to} s_{22}$$

$$X_{11,21}^{(T)}(|A_{11}|) \underset{|A_{11}|\to 0}{\to} 0$$

$$X_{21,21}^{(T)}(|A_{11}|) \underset{|A_{11}|\to 0}{\to} 0$$

X-Parameters Collapse to S-Parameters in Linear Systems

$$b_{i,k} = \sum_{j,l} \left(X^{S}_{ij,kl}(|a_{11}|) P^{k-l} \cdot a_{j,l} + X^{T}_{ij,kl}(|a_{11}|) \;\; P^{k+l} \cdot a^{*}_{j,l} \right)$$

Assume linear ($X^T = 0$)

$$b_{i,k} = \sum_{j,l} \left(X^{S}_{ij,kl} P^{k-l} \cdot a_{j,l} \right)$$

Assume fundamental frequency only (k = l = 1)

$$b_{i} = \sum_{j} \left(X^{S}_{ij} \cdot a_{j} \right)$$

Assume 2 port (i and j = 1 -> 2)

$$b_{i=1->2} = \sum_{j=1}^{2} \left(X^{S}_{ij} \cdot a_{j} \right)$$

$$b_1 = X^{S}_{11} \cdot a_1 + X^{S}_{12} \cdot a_2$$
$$b_2 = X^{S}_{21} \cdot a_1 + X^{S}_{22} \cdot a_2$$

$$b_1 = S_{11}a_1 + S_{12}a_2$$
$$b_2 = S_{21}a_1 + S_{22}a_2$$

- **Definitions**

  i = output port index

  j = input port index

  k = output frequency index

  l = input frequency index